Die SOPHISTen

Gottlob Frege

Gottlob Frege is generally regarded as the father of modern analytic philosophy. His efforts led directly to later investigations of logic and language by Russell, Wittgenstein, the Vienna Circle and uncountable numbers of later analytic/linguistic philosophers. Frege single-handedly set the goal of rigorous proof for mathematics and philosophy higher than anyone since Euclid.

SOPHIST values Frege's pioneering writings because he dealt with the exact same problems encountered in modern system design and requirements engineering. We can only be glad that the mindscape was explored by Frege over one hundred years ago as he scouted out the territory and pointed out the dangerous areas for later thinkers to further investigate.

Frege (8 Nov 1848- 25 July 1925) stated in the last year of his life that, „…a great part of the work of a philosopher consists – or at least ought to consist – in a struggle against language.“ (p369, Frege Reader [Sources of Knowledge of Mathematics]) Perhaps he of all people would take such an extreme opinion since the illusions of language had blinded even he to the point that the original goal of all his work on the foundations of mathematics was nullified through an appealing illusion of language.

The foundations of mathematics were of great concern for the mathematicians in Frege's day because all of mathematics rested upon the consistency of kindergarten arithmetic. Like a constructed tower to the heavens, each mathematical theory rested upon previous theories, and the cornerstone at the bottom of this edifice to the gods was simple arithmetic. 0+1+2=3?

Some felt that numbers were shortcuts invented in ancient times to discuss quantity. For example, suppose caveman Grok and cavewoman Urke have a small pile of stones. Without numbering the stones, I can only say that Grok has as many hands as Urke and when Grok and Urke both take a stone in each hand, there are no more stones left in the pile. Since Grok and Urke were anatomically similar to homo sapiens, we modern humans would say they had a total of four stones between them. Needles to say numbers are a tremendous shortcut in communication. Try doing without numbers for an hour!

Frege found the above, „gingerbread or pebble arithmetic,“ very lacking. He compared it to studying America by taking, „…[oneself] back to the position of Columbus as he caught the first dubious glimpse of his supposed India.“ Frege observed that numbers can describe abstractions, such as „three strokes of a clock,“ or „three ways to solve the equation“ (p94) so numbers would seem to cover more than what is merely physical!

Frege felt that experience may be necessary to learn the laws of arithmetic, like cavemen with their stones, but that does not mean that number is derived from the physical world. The physical world in some sense seems to 'draw out' knowledge that we learn to access, although the lesson may seem very simple at first glance. For example, Newton's apple falling to the ground ultimately spawned a theory of planetary motion, mysterious gravitational action-at-a-distance and the calculus! „Experience may be required to learn truth,“ (p96) but the truth is out there.

Furthermore, number is most unlike any other description language may enable. For instance, consider the assertions Solon was wise and Thales was wise. The conclusion is then Solon and Thales were wise. Now try the same logic replacing wise with the number one. Solon was one. Thales was one. Solon and Thales were one. (p97, Frege Reader [Die Grundlagen der Arithmetik]) What is going on here? Because of examples like this, Frege felt „The rules of number stand in the most intimate connection with those of thought.“ (p95, FR [Grundlagen]) and furthermore, „Arithmetic governs the realm of the numeralbe, actual, intuitable, everything thinkable! (p95, FR [Grundlagen])

In Frege's efforts to deduce all of mathematics from logic, he had developed a handful of definitions and rules of inference. Starting with '0' and '1' and a relation between them, he defined all further numbers. He demonstrated the relation between numbers and what they represent. Frege also included notions of truth and false to make his thinking complete. It seemed as though the most abstract mathematics had been reduced to logic. Then lighting struck.

In 1902, Bertrand Russell found a boundary in the form of a question beyond which language and the human mind tread, but logic does not. After stripping away all the notations, definitions and theory, a simplified form of the question is as follows: „Is the statement This sentence is false itself true or false?" If the statement is true, then it is immediately false! If is false, then it is immediately true! Is it true? False? Both? Neither? Similarly, „My plan is to have no plan," or „No rule is the best rule" are also logically paradoxical. Yet the statements make sense from certain perspectives.

Near the end of his life, Frege summarized Russell's paradox problem. „One feature of language that threatens to undermine the reliability of thinking is its tendency to form proper names to which no [physical/mental] objects correspond." For example, the sentence What is the applicability of the concept comfortable chair? The problem lies with the definite article the. The seems to make applicability into a reference to a physical object such as the car or the basketball! Yet the applicability in this sentence is not an object or collection of objects whatsoever, but merely a reference to the process of classifying physical objects as being comfortable chairs! Language has but few tools with which it operates and in this case, for lack of a better option, it seems to require the hammering of a very square peg into a very non-square hole.

„It is extremely difficult, perhaps impossible, to test every expression offered us by language to see whether it is logically innocuous. So a great part of the work of a philosopher consists – or at least ought to consist – in a struggle against language. But perhaps only a few people are aware of the need for this." (p369, FR [Sources of Knowledge of Mathematics]) „The difficulties which this idiosyncrasy of language entangles us in are incalculable." (p370, ibid) „Such experiences teach us how necessary it is to place the strictest demands on precision of spoken and written forms of expression." (p210-211, FR [Grundgesetze der Arithmetik]) „Work in logic just is, to a large extent, a struggle with the logical defects of language, and yet language remains for us an indispensable tool." (p323,FR [My Basic Logical Insights])

Frege was never to answer Russell's question. But Frege did make some excellent observations about the nature of language and logic that we best not forget. He came as close as anybody to build a mathematical foundation upon logic, but as he neared his fountain of truth in the jungle of language, the illusion became apparent. Language was not to reveal her secrets so easily, even to as rigorous and precise a thinker as Gottlob Frege.

SOPHIST recognizes Frege as the modern founder of the dissection of language in a formal and rigorous fashion. Frege was the first modern to directly recognize the slipperiness of language. He also made some very astute observation regarding the dangers and tendencies of language to intertwine with thought, ever so subtly, such that at the borders of consciousness, the clearly defined edges of definition would fray.

Source: The Frege Reader, ed. Michael Beaney. Oxford: Blackwell, 1997. A excellent translated collection of Frege's letters and major works including Begriffschrift (Conceptual Notation), Grundlagen der Arithmetik (The Foundations of Arithmetic), Sinn und Bedeutung (Sense and Reference), Grundgesetze der Arithmetik (Basic Laws of Arithmetic) and various letters and personal journals.